General Equilibrium Effects of the Minimum Wage*

Ryo Hasumi[†1,2] and Tetsuaki Takano[‡1]

[1]Faculty of Economics, Musashi University

[2]Japan Center for Economic Research

January 12, 2024

Musashi University Discussion Paper Series No. 104

Abstract

We examine the impact of a large minimum wage increase on macroeconomic outcomes by developing a search-matching model with heterogeneous households and an endogenously determined unemployment rate. The model is calibrated to replicate the U.S. economy, allowing us to simulate minimum wage increases. We find negative non-linear effects of a substantial nationwide minimum wage increase on macroeconomic outcomes, including employment. In the $15 minimum wage scenario, output falls by approximately 12% relative to the baseline while the unemployment rate increases to 9.6% from 5% in the baseline. The net job loss is relatively small during the transition path immediately after a minimum wage increase, which has implications for the appropriate design of empirical analyses using natural experiments.

Keywords: Minimum Wage, Search and Matching, Heterogeneous Agents, Mean Field Game

JEL Codes: E24, E6, J64

*This work was supported by JSPS KAKENHI 19K23232, 23K01444.

†Musashi University; 1-26-1 Toyotamakami, Nerima-ku, Tokyo, 176-8534, Japan.
Email: ryo.hasumi@u.musashi.ac.jp

‡Email: tetsuaki.takano@gmail.com

1 Introduction

The minimum wage has attracted interest recently as a means of supporting low-income workers. What distinguishes it from income redistribution policies is its very nature, requiring no financial resources. A growing movement in the United States (U.S.) seeks to raise the current federal minimum wage of $7.25 to at least $15 to help increase low-income workers' wages.[1] While some states already have a minimum wage of $15, the Current Population Survey (CPS) shows that some workers are still paid less than $7.25 per hour (see Figure 4).

Existing literature has raised concerns about the adverse employment effects of minimum wage increases (e.g., Neumark and Wascher [2007]). Meanwhile, recent influential empirical studies report no negative employment effects for actual minimum wage increases (e.g., Cengiz et al. [2019], Dustmann et al. [2022], and Engbom and Moser [2022]). These studies estimate the impact of minimum wage increases through natural experiments employing microdata such as administrative data.[2] However, note that the scope of natural experiments may not fully encompass large-scale minimum wage increases, such as from $7.25 to $15, especially those at the national (as opposed to local) level. Moreover, minimum wages are often increased after wage growth. Natural experiments may not measure the impact of exogenous policy implementation.

A nationwide minimum wage increase (federal in the U.S. or intra-regional in the EU) applied to an entire area of free labor mobility has a completely different meaning from a local minimum wage increase. We have no way to measure the impact of a large nationwide minimum wage increase except by developing and simulating a macroeconomic model.

Because the minimum wage affects only low-productivity employees (except for general equilibrium effects), how the wage distribution is reproduced in the model is crucial. In the seminal study by Flinn [2006], who investigated the minimum wage using a search-matching model, wages are randomly drawn from a fixed distribution during the matching process while ignoring household skills or productivity. The model in this study follows Flinn [2006] but allows wage bargaining based on sticky and idiosyncratic productivity

[1] Under the Biden administration, Democratic lawmakers announced specific legislation to raise the minimum wage to $15 on January 26, 2021. This legislation, known as the Raise the Wage Act of 2021, proposes to initially raise the minimum wage to $9.50 on the date of enactment and then incrementally increase it to $15 over the next four years. Once it reaches $15, it would be increased each year by indexing it to the median hourly wage for all employees.

[2] Wiltshire et al. [2023] reported no decrease in employment even when the minimum wage was raised to $15, although the analysis is limited to California and New York.

rather than being randomly determined from some distribution. Recently, Berger et al. [2022] and Hurst et al. [2022] simulated a scenario where the minimum wage increases to $15. However, few empirical studies refer to the labor supply monopsony on which these two studies theoretically rely (Azar et al. [2019] is an exception). Crucially, the conclusion that raising the minimum wage could have positive employment effects, which stems from the monopsony assumption, goes beyond extant empirical findings.

Notably, most models addressing unemployment or the minimum wage assume a family that can share the risk of unemployment (e.g., Merz [1995], Shimer [2005], Gertler and Trigari [2009], Berger et al. [2022], and Hurst et al. [2022]). However, this assumption is not only unrealistic but also underestimates the risk of unemployment and undervalues the role of social security systems, including the minimum wage. Without assuming a family, Huggett [1993] proposed a heterogeneous agent model with unemployment, and Alonso-Ortiz and Rogerson [2010] applied this model to replicate the wage distribution. We extend these models to a search-matching model in which the transition from unemployment to employment is not random; instead, the probability of transitioning to employment is determined by the convex adjustment cost of new hires, as in Gertler and Trigari [2009]; this depends on the wage or productivity of each household.[3] In addition, to examine the effect on labor force participation, we follow Flinn [2006] and adopt a three-state model: employment, unemployment, and out of the labor force (out-of-labor).[4]

Our theoretical contribution is the development of a search-matching model with heterogeneous households in which the unemployment rate is determined endogenously without assuming a representative household or family. As most studies have raised concerns about the negative employment effects of minimum wage increases, our study provides a perspective on the underlying mechanism. We calibrate the model to replicate the U.S. economy and analyze the macroeconomic impact of, for example, the $15 minimum wage. Furthermore, we evaluate not only the long-term (steady-state) effects, but also the short-term effects along the transition path to the steady state.

This study has three main findings. First, contrary to the recent literature that expects limited employment effects from minimum wage increases, we find a broad impact of a large nationwide minimum wage increase on macroeconomic outcomes, including employment. In

[3]Bagger et al. [2014] used Danish microdata, specifically matched employer-employee data, to conduct a structural estimation of an equilibrium job search model. The authors showed that the job-finding rates are positively correlated to individual productivity.

[4]To identify the state of out-of-labor, we assume that households in this state can gain utility from home production.

the \$15 minimum wage scenario, output falls by approximately 12% relative to the baseline, while the unemployment rate rises to 9.6% from 5% in the baseline. Households with no employment prospects move to out-of-labor, and the labor force participation rate falls to 62.0% from 77.6% in the baseline. As the transition of low-productivity households from employment to unemployment or out-of-labor occurs from the left tail of the wage distribution, the impact increases nonlinearly with the size of the minimum wage hike.

Second, as a theoretical and visually illustrated contribution, we demonstrate the existence of excess and missing jobs, as proposed by Cengiz et al. [2019] (see Figure 8). Meanwhile, we do not replicate the finding of Flinn [2006] that an increase in the value of search because of a minimum wage hike causes increases in the value of unemployment and labor force participation rate. This is because, in our model, a minimum wage increase raises costs for firms, which lowers job-finding rates through a decline in the value of filled jobs. However, even in Flinn [2006], an increase in the participation rate is observed only with exogenous contract rates. Our finding is consistent with Cengiz et al. [2019], in that a minimum wage increase does not necessarily cause an increase in the labor force participation rate.

Finally, the dynamic simulation shows that a temporary increase in the real minimum wage causes the unemployment rate to overshoot relative to the long-term equilibrium. Moreover, on the transition path immediately after a minimum wage increase, the net job loss is relatively small because excess jobs are created to partially compensate for missing jobs. Thus, our result suggests that empirically observing a significant reduction in employment immediately after a minimum wage increase may be difficult.

The rest of the paper is organized as follows. The next section briefly reviews the related literature. In Section 3, we set up the model and calibrate the model parameters in Section 4. Section 5 presents the simulation results, distinguishing between the steady state and transition path. In Section 6, we discuss the differences in the results compared with those of relevant studies. Finally, Section 7 presents the conclusion of this study.

2 Related literature

Since Card and Krueger [1994] reported that minimum wage increases have no negative effect on employment using a difference-in-differences (DID) design, various cases and data have been examined to investigate the causal effect of the minimum wage. Neumark and Wascher [2007] surveyed more than 100 empirical studies and found that two-thirds of the articles suggest a negative effect of the minimum wage on employment, while about 10 articles suggest positive effects. Furthermore, of the 33 articles considered credible by the authors, 28 implied negative effects and concluded that raising the minimum wage negatively affects employment. This finding remains unchanged in Neumark and Shirley [2022], who summarized the key estimates that support the conclusions of each published study. The authors found that in the U.S., the negative employment effects of the minimum wage are larger for less-skilled workers and no evidence suggests that the estimates have become less negative in more recent studies.[5]

Besides the employment effects of the minimum wage increase, Giuliano [2013] and Cengiz et al. [2019] investigated the impact of minimum wage increases on labor force participation and wage distribution. The former examined the impact of the 1996 federal minimum wage increase using personnel data from a large U.S. retail firm with more than 700 stores. While the minimum wage increase had a negative, although statistically insignificant impact on overall employment, the relative wage increases for teenagers working just above the minimum wage increased employment, particularly for those from wealthier households. Specifically, the employment ratio of teenagers increased by 0.6–0.9 percentage points because of the event, and this effect was both significant and robust. The underlying mechanism had been unclear in the conventional view. However, the authors found that the minimum wage increase caused an increase in labor force participation, thereby contributing to employment growth.

Until recently, few studies have examined the impact of minimum wage increases on the entire wage distribution. Using CPS data from 1979 to 2016, Cengiz et al. [2019] employed a DID design to estimate the impact of a minimum wage increase on the wage distribution. Within five years of a minimum wage increase, employment at or below the minimum wage decreased significantly and substantially, whereas employment above the minimum wage increased. However, total employment remained unchanged, and only hourly wages for the same job increased. Consequently, the wages of the workers affected by the minimum wage increase increased by approximately 6.8%. Moreover, employment in the upper part of the wage distribution did not change and the spillover effect was limited to +$3 from the new minimum wage, which was consistent with Autor et al. [2016]. However, the aforementioned study found no evidence that low-skilled employment was replaced by high-skilled employment because of this event.

Meanwhile, Engbom and Moser [2022], who examined the labor market in Brazil from

[5]Neumark [2018], Neumark [2019], and Manning [2021] are recent survey articles.

1996 to 2018 using administrative data and household surveys, found that raising the minimum wage had a spillover effect even on the upper part of the wage distribution, accounting for 45% of the reduction in income inequality. The authors incorporated unobserved worker heterogeneity, the minimum wage, and endogenous job creation into the search-matching model of Burdett and Mortensen [1998], which introduced an on-the-job search into a random search model, and estimated the parameter values to explain the empirical results. The authors noted one factor in the spillover effect: firms paying wages even above the minimum wage had the incentive to raise wages to maintain their own rank in the wage distribution. They also found a slightly negative impact on employment. While low-productivity firms reduced job creation owing to lower profit margins from the minimum wage increase, high-productivity firms increased hiring. The reallocation effect limited the decline in the aggregate employment rate to 0.7%.

Similarly, Dustmann et al. [2022] used administrative data to analyze the impact of the nationwide introduction of a uniform minimum wage in Germany in 2015. The authors concluded that 15% of employment was exposed to the minimum wage, but no effect on employment was observed. Specifically, the policy caused the reallocation of employment from small to large firms, and low- to high-paying firms; at the regional level, the average quality of firms improved in the more severely affected regions. Moreover, the reallocation effect accounted for 17% of the wage increase caused by the event.

Next, we briefly summarize studies using structural models. Flinn [2006] is a seminal study that examined the relationship between employment, unemployment, and the minimum wage from both theoretical and empirical perspectives. Theoretically, based on the McCall search model, the author demonstrated that a minimum wage increase raises the unemployment rate and can increase the labor force participation rate when the value of search increases. Empirically, the author used the minimum wage increase from \$4.25 to \$5.15 in September 1997 in the U.S. as a natural experiment and evaluated the optimal level of the minimum wage based on social welfare using data such as the CPS. Notably, the results vary depending on assumptions, and the optimal level can be higher or lower than the actual minimum wage.

More recently, Hurst et al. [2022] and Berger et al. [2022] examined the effects of the minimum wage using macroeconomic models. Both studies are based on a monopsony model in which employers have the dominant power to determine employment and wages. In the model of Hurst et al. [2022], unemployment exists because of the friction caused by the directed search by dominant firms. Households consist of two types: non-college and college graduates. For each household type, productivity follows a log-normal distribution. However, the study only considered a steady state with zero net saving. Berger et al. [2022] assumed firm heterogeneity, and wages had a distribution because the number of firms in the market and the productivity of each firm differ. The model has two representative households, one each for non-college and college graduates, and does not include unemployment because it is not a search model. Both models have wage markdowns owing to firms' monopsony power; therefore, raising the minimum wage can improve employment and social welfare.

In Section 6, we compare the results of these studies using models, including Engbom and Moser [2022], with ours.

3 Model description

We assume heterogeneous households as in Aiyagari [1994], Huggett [1993], or Achdou et al. [2022], whose state variables are asset holdings a, log productivity z, and employment status h. Specifically, we define the lifetime utility of households, processes of asset accumulation, and evolution of log productivity as follows:

$$
\begin{aligned}
U_t &= \int_0^\infty \exp(-\rho t) u(c_t, \gamma(h_t)) dt, \\
da_t &= [e_t(z_t, h_t) - \tau_w(e_t(z_t, h_t)) \mathbb{1}_{h=e} + r_t a_t - c_t] dt = s_t dt, \;\; a_t \geq \underline{a}, \\
dz_t &= -\eta_z z_t dt + \sigma dB_t, \qquad (1)
\end{aligned}
$$

where c_t is consumption, s_t is savings, and r_t is the real interest rate. $\gamma(\cdot)$ is home production, $e_t(\cdot, \cdot)$ is earnings, and $\tau_w(\cdot)$ is the progressive income tax amount defined below.[6] B_t is a Wiener process; therefore, the log productivity process is an Ornstein-Uhlenbeck process with damping parameter η_z and volatility parameter σ. Employment status h is either employed e, unemployed u, or out of the labor force (out-of-labor) o. Depending on h, home production $\gamma(\cdot)$ and earnings $e_t(\cdot, \cdot)$ take the following values:

$$
\gamma(h_t) = \begin{cases} 0 & (h_t = e, u) \\ \bar{\gamma} & (h_t = o) \end{cases},
$$

and

$$
e_t(z_t, h_t) = \begin{cases} w_t(z_t) & (h_t = e) \\ b(z_t) & (h_t = u, o) \end{cases},
$$

[6] The assumption that households directly gain utility from home production is found, for example, in Greenwood and Hercowitz [1991], Benhabib et al. [1991], or Veracierto [2008].

where $w_t(z_t)$ and $b(z_t)$ are the wage and social security benefits received by a household with log productivity z_t. We specify the functional form of the utility function $u(\cdot)$, progressive income tax $\tau_w(\cdot)$, and social security benefits $b(\cdot)$ as follows:

$$u(c,\gamma) = \frac{(c+\gamma)^{1-\theta}-1}{1-\theta},$$
$$\tau_w(w) = w - (1-\bar{\tau}_w)(w/\mathrm{E}[w])^{1-\tau}\mathrm{E}[w],$$
$$b(z) = [1 + \mathbb{1}_{w(z)\geq w_b \mathrm{Q}_{0.5}[w]}(w_b \mathrm{Q}_{0.5}[w]/w(z) - 1)]w(z)\bar{b},$$

where $\mathrm{E}[w]$ and $\mathrm{Q}_{0.5}[w]$ are the mean and median wages, respectively.[7] The wage rate $w_t(z_t)$ is determined to maximize the Nash product of the household and firm matched surpluses, as described below.

Let $g_t(a,z,e)$ be the density of each state of households. By definition, $\iint[g_t(a_i,z_j,e) + g_t(a_i,z_j,u) + g_t(a_i,z_j,o)]didj \equiv 1$, where we set $a_i = i$ and $z_j = j$. The employment process is determined by each household's log productivity, z_t. The ratios of employment, unemployment, and out-of-labor in the households with $z = j$, that is, $g_{jt} = \int[g_t(a_i,z_j,e) + g_t(a_i,z_j,u) + g_t(a_i,z_j,o)]di$, are defined by $n_{jt} = g_{jt}^{-1}\int g_t(a_i,z_j,e)di$, $u_{jt} = g_{jt}^{-1}\int g_t(a_i,z_j,u)di$, and $o_{jt} \equiv 1 - n_{jt} - u_{jt}$, respectively. The job-finding rate for households, ζ_{jt}, can be expressed by:

$$\zeta_{jt} = \zeta_t(z_j) = m_{jt}/u_{jt},$$

with matches (new hiring) $m_{jt} = m(u_{jt},\nu_{jt})$ where ν_{jt} represents job vacancies at firms. We also assume that the job separation rate is constant at q. The Hamilton-Jacobi-Bellman (HJB) equations for each household status are as follows:

$$\begin{aligned}
\rho v_t^e(a_t,z_t) &= \max_c[u(c_t,0) + \mathcal{A}v_t^e(a_t,z_t) + \partial_t v_t^e(a_t,z_t)] + q[v_t^u(a_t,z_t) - v_t^e(a_t,z_t)],\\
\rho v_t^u(a_t,z_t) &= \max_c[u(c_t,0) + \mathcal{A}v_t^u(a_t,z_t) + \partial_t v_t^u(a_t,z_t)]\\
&\quad + \mathbb{1}_{v_t^u\geq v_t^o}\zeta_t(z_t)[v_t^e(a_t,z_t) - v_t^u(a_t,z_t)] + \mathbb{1}_{v_t^u< v_t^o}[v_t^o(a_t,z_t) - v_t^u(a_t,z_t)],\\
\rho v_t^o(a_t,z_t) &= \max_c[u(c_t,\bar{\gamma}) + \mathcal{A}v_t^o(a_t,z_t) + \partial_t v_t^o(a_t,z_t)] + \mathbb{1}_{v_t^u\geq v_t^o}[v_t^u(a_t,z_t) - v_t^o(a_t,z_t)],
\end{aligned}$$

where $\mathcal{A} = -\eta_z z\frac{\partial}{\partial z} + \frac{1}{2}\sigma^2\frac{\partial^2}{\partial z^2} + s\frac{\partial}{\partial a}$. $\mathbb{1}_{v_t^u\geq v_t^o}$ is an indicator function that returns 1 if the value of unemployment is greater than that of out-of-labor, implying that households choose a

[7] The functional form of the progressive income amount follows that of Feldstein [1969], which is widely used in studies of taxation in heterogeneous agent models, such as Heathcote et al. [2017]. The functional form of social security benefits implies a decreasing replacement rate, which is commonly applied to unemployment benefits in the U.S. states. The adopted parameter values are described in Section 4.

state of greater value from unemployment and out-of-labor, and the transition occurs at a rate of 1 per unit time.

Once the HJB equations are given, the corresponding Kolmogorov forward equation determines the transition of households' density $g_t(a, z, e)$ (Achdou et al. [2022]). We define aggregate effective labor as:

$$L_t = \left[\int \left(e^{z_j} n_{jt}\right)^{\frac{\varepsilon-1}{\varepsilon}} g_j dj \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where ε represents the elasticity of substitution of effective labor. The aggregate capital stock is given by:

$$K_t = \sum_h \iint a_i g_t(a_i, z_j, h) di dj.$$

We assume that firms are homogeneous with a constant returns to scale production function $F(K, L) = K^\alpha L^{1-\alpha}$ and that the productivity distribution of workers employed at time t is identical across firms. Let the firm's hiring rate $x_{jt} = \frac{\zeta_{jt} u_{jt}}{n_{jt}}$, and let the adjustment cost of employment be $\phi(x_{jt})n_{jt}$. Then, the firm's value of the filled job with $z = j$, $f_{jt} = f_t(n_{jt}, z_j)$ satisfies the following HJB equation:

$$\begin{aligned} \rho \int f_{jt} g_{jt} dj &= F(K_t, L_t) - (r_t + \delta) K_t \\ &+ \int \{\max_x [-w_{jt} n_{jt} - \phi(x_{jt}) n_{jt} + (x_{jt} - q) n_{jt} \partial_n f_{jt}] - \eta_z z_j \partial_z f_{jt} + \frac{1}{2}\sigma^2 \partial_z^2 f_{jt} + \partial_t f_{jt}\} g_{jt} dj, \end{aligned}$$

where the constraint $x_{jt} \in [0,\ \min\{q + \rho,\ q + u_{jt}/n_{jt}\}]$ is imposed such that f_{jt} is finite, and the relationship $dn_{jt} = (x_{jt} - q)n_{jt} dt$ is applied. When g_{jt} is in a steady state, the first-order condition and envelope theorem imply that:

$$\phi'(x_{jt}) = \partial_n f_{jt},$$

$$\rho \partial_n f_{jt} = \mu_{jt} - w_{jt} - \phi(x_{jt}) + (x_{jt} - q)\partial_n f_{jt} - \eta_z z_j \partial_z \partial_n f_{jt} + \frac{1}{2}\sigma^2 \partial_z^2 \partial_n f_{jt} + \partial_t \partial_n f_{jt},$$

where $\mu_{jt} = (1 - \alpha) \left(\frac{K_t}{L_t}\right)^\alpha e^{z_j} \left(\frac{e^{z_j} n_{jt}}{L_t}\right)^{-\frac{1}{\varepsilon}}$ is the marginal product of labor. Next, we specify the functional form of employment adjustment cost as $\phi(x) = \frac{1}{2}\kappa x^2$.

For each productivity level $z = j$, the wage rate before the minimum wage is applied, $\tilde{w}$, is assumed to be determined by maximizing the Nash product:

$$\Theta_{jt} = V_t(\tilde{w}_{jt})^\eta J_t(\tilde{w}_{jt})^{1-\eta},$$

where the matching surpluses of households and firms, $V_t(\cdot)$ and $J_t(\cdot)$, are defined below. The actual wage is determined by applying the minimum wage w^* to $\tilde{w}$; that is:

$$w_t(z_j) = \max\{\tilde{w}_{jt},\ w_t^*\}.$$

Assume that the household's value of employment V_{jt}^e and unemployment V_{jt}^u satisfy the following equations:

$$\rho V_{jt}^e = \tilde{w}_{jt} + q(V_{jt}^u - V_{jt}^e),$$
$$\rho V_{jt}^u = b(z_j) + \zeta_{jt}(V_{jt}^e - V_{jt}^u),$$

then we obtain the following relationship:

$$\rho V_t(\tilde{w}_{jt}) = \tilde{w}_{jt} - b(z_j) - (q + \zeta_{jt})V_t(\tilde{w}_{jt}),$$

with the household surplus $V_t(\tilde{w}_{jt}) \equiv V_{jt}^e - V_{jt}^u$.[8] Similarly, assume that the firm's values of filled jobs and vacancies satisfy the following equations:

$$\rho J_{jt}^e = \mu_{jt} - \tilde{w}_{jt} + q(J_{jt}^\nu - J_{jt}^e),$$
$$\rho J_{jt}^\nu = \zeta_{jt}\theta_{jt}^{-1}(J_{jt}^e - J_{jt}^\nu),$$

then we have the following relationship:

$$\rho J_t(\tilde{w}_{jt}) = \mu(z_j) - \tilde{w}_{jt} - (q + \zeta_{jt}\theta_{jt}^{-1})J_t(\tilde{w}_{jt}),$$

with the firm surplus $J_t(\tilde{w}_{jt}) \equiv J_{jt}^e - J_{jt}^\nu$ and labor market tightness $\theta_{jt} = \nu_{jt}/u_{jt}$.[9] With these assumptions, the wage rate before the minimum wage is applied, $\tilde{w}$, must satisfy the following equation:

$$\eta(\mu_{jt} - \tilde{w}_{jt}) - (1 - \eta)(\tilde{w}_{jt} - b(z_j)) = 0.$$

The real interest rate equals the marginal productivity of capital:

$$r_t = \alpha \left(\frac{K_t}{L_t}\right)^{\alpha - 1} - \delta.$$

[8]Here, we assume that household asset holdings do not affect wages; to our knowledge, few cases suggest employees' asset holdings affect wage bargaining.

[9]In this model, the cost of posting vacancies is assumed to be zero as in Gertler and Trigari [2009] or Shimer [2010].

The aggregate profit of the firms is given by:

$$\Pi_t = K_t^{\alpha} L_t^{1-\alpha} - W_t - (r_t + \delta) K_t - \Phi_t,$$

with the sum of wages $W_t = \iint w_t(z_j) g_t(a_i, z_j, e) didj$ and sum of hiring costs $\Phi_t = \iint \phi(x_{jt}) g_t(a_i, z_j, e) didj$. We assume that the government collects corporate profits in addition to the progressive income tax and provides government consumption, which is determined as the residual of the balanced budget and social security benefits. The government's budget constraint is then given by:

$$G_t = \iint \tau_w(w_t(z_j)) g_t(a_i, z_j, e) didj - \iint b(z_j)[g_t(a_i, z_j, u) + g_t(a_i, z_j, o)] didj + \Pi_t.$$

Finally, the market-clearing conditions are:

$$Y_t = K_t^{\alpha} L_t^{1-\alpha} - \Phi_t,$$
$$dK_t = (Y_t - C_t - \delta K_t - G_t) dt,$$

with aggregate consumption, $C_t = \sum_h \iint c_t(a_i, z_j, h) g_t(a_i, z_j, h) didj$.

4 Calibration

4.1 Parameter settings

We calibrate the parameter values of the model to replicate the U.S. economy, focusing on reproducing the labor market. First, the unit of time t is set to be monthly; in countries with relatively high job-finding and separation rates, such as the U.S., a worker's status may change more than once during a period if a longer time unit is used.

Table 1 shows the parameter values. Relative risk aversion θ, capital share α, and the lower bound of asset holdings $\underline{a}$ are set to 2.0, 0.33, and 0, respectively, following the literature (e.g., Achdou et al. [2022] and Gertler et al. [2008]). The discount rate ρ is set to 0.0068 to match the capital-output ratio (K/Y) to 3.0 on an annual basis. Home production $\bar{\gamma}$ is set to 0.52 to match the labor participation rate of 0.773. The capital depreciation rate, δ, is set to 0.0043 (annual rate of 0.052). The job separation rate q is set to 4% per month, as estimated from the Job Openings and Labor Turnover Survey. The coefficient of the employment adjustment cost κ is set to 415 to obtain a steady-state unemployment rate of 5%. The labor elasticity of substitution ε is set to 5.545, as estimated from the dynamic stochastic general equilibrium (DSGE) model by Galí et al. [2012]. The bargaining

Parameter	Description	Value	Source/Target
ρ	Discount rate	0.0068	Capital-output ratio
θ	Relative risk aversion	2.0	Achdou et al. [2022]
$\bar{\gamma}$	Home production	0.52	Labor participation rate
$\underline{a}$	Borrowing limit	0	Achdou et al. [2022]
α	Capital share	0.33	Gertler et al. [2008]
δ	Capital depreciation rate	0.00433	Data
q	Separation rate	0.04	Data
κ	Labor adjustment cost	415	Unemployment rate
ε	Substitute elasticity for labor	5.545	Galí et al. [2012]
η	Bargaining power for workers	0.73	Labor income share
η_z	Persistence of idiosyncratic shocks	0.0025	Wage distribution
σ_z	Standard deviation of shocks	1.05	Wage distribution
$\overline{\tau}_w$	Marginal tax rate for average labor income	0.1807	CBO [2019]
τ	Progressivity of labor income tax	0.181	Heathcote et al. [2017]
$\bar{b}$	Social security benefits replacement rate	0.5	Institution
w_b	Threshold for replacement rate	1.23	Institution
w^*	Minimum wage	0.107	See text

Table 1: Parameter values for calibration

weight of workers η is set to 0.73 to obtain a labor income share of 0.54. We select the target moments referring to the long-term average of the U.S. economy. The persistence and variance of the idiosyncratic shock to productivity faced by households are set to 0.0025 and 1.05, respectively, to approximate the actual distribution of the hourly wage, which is constructed by dividing weekly earnings by weekly hours worked using microdata from the 2019 CPS. Table 2 lists the target moments and corresponding model outcomes.[10]

The coefficients of progressive income tax (upper panel of Figure 1) and social benefits (lower panel of Figure 1) are determined as follows. The degree of progressivity of the labor income tax rate, τ, is set to 0.181 based on Heathcote et al. [2017]. The shift parameter $\overline{\tau}_w$ is set to 0.1807 to replicate the average marginal tax rate of 32.9% (sum of individual income tax, 21.9%, and payroll tax, 11.0%) of U.S. labor income in 2018, as estimated by

[10]Regarding grids used in the finite difference method, we set up a lattice consisting of 101 nodes from 0 to 400 at intervals of 4 for asset a and 101 nodes from -5 to 5 at intervals of 0.1 for log productivity z.

Moment	Target	Model	Source
Capital-output ratio	36.0	35.946	SNA
Labor participation rate	0.773	0.7756	CPS
Unemployment rate	0.05	0.0504	CPS
Labor income share	0.54	0.5405	SNA
Std of log wage	0.604	0.6027	CPS

Table 2: Target moments

Note: Target indicates data for calibration and Model indicates the corresponding model outcomes.

the Congressional Budget Office (CBO [2019]). The replacement rate for unemployment benefits varies somewhat by state; still, some commonality exists as it remains constant up to a certain level of pre-unemployment earnings and decreases thereafter. Here, we set the constant replacement rate $\bar{b}$ and threshold w_b to 0.5 and 1.23, respectively, referring to the regulations in California, where the upper limit and payment period of unemployment benefits are standard in the U.S.[11] In other words, the replacement rate is 50% up to 1.23 times the median of the wage distribution; beyond that, the level remains constant and inversely proportional to the expected wage. The minimum wage w^* is set to 0.107, which is the tipped minimum wage of $2.13 standardized by the median wage.

4.2 The results of the baseline calibration

Figure 2 illustrates the ratio of employed, unemployed, and out-of-labor households at each productivity level in the steady state, with the logarithm of productivity on the horizontal axis. If households are highly productive, they will always be employed. However, those with low productivity prefer to be out-of-labor because the sum of home production and social security benefits exceeds the expected wages. The higher the productivity, the more profit firms earn; therefore, they pay large search costs to hire workers. Job separation occurs with exogenous probability q; however, if productivity is sufficiently high, households will find jobs immediately without being unemployed. Therefore, in a high-productivity region, a firm's hiring rate is maintained at the job separation rate level (upper panel of Figure 3).

Unemployment occurs at productivity levels slightly below the median. In a very low

[11] The benefit cap in California is $450 per week, which is about the same as the $441 per week average value across all 50 states and the District of Columbia. Besides, the duration cap of 26 weeks is close to the average of 24.4 weeks and mode of 26 weeks.

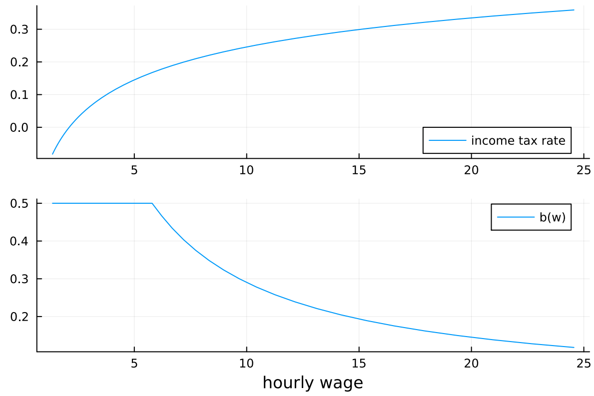

Figure 1: Progressive income tax and income replacement rates for social benefits

Note: The top panel shows income tax rates and bottom panel shows income replacement rates for social benefits. The functional forms are described in Section 3.

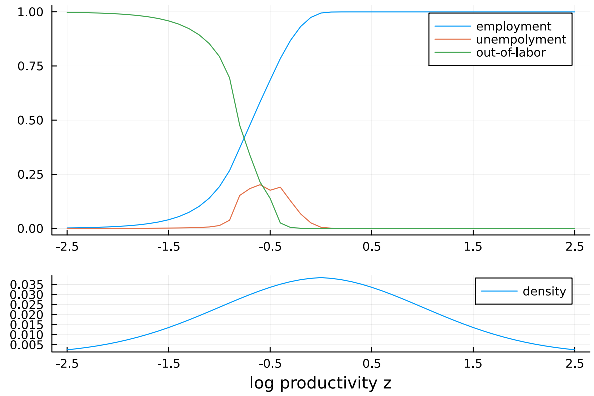

Figure 2: Distribution of employment, unemployment, and out-of-labor

Note: The top panel shows the ratio of employed, unemployed, and out-of-labor at each productivity level. The bottom panel shows the probability density of log productivity for each household in the steady state.

productivity region, the hiring rate x of firms is much lower than the separation rate and households choose out-of-labor over unemployment to obtain home production. As productivity increases, the hiring rate approaches the separation rate, and households gradually become employed as they choose unemployment over out-of-labor. The ratio of unemployment is the highest in the region where the hiring rate is slightly lower than the highest point. However, workers exist even in a region with no unemployment because some employees become less productive after being hired, and the destruction of the match occurs exogenously with probability q.

In the baseline calibration, the minimum wage is set to \$2.13, which is the same as the tipped minimum wage. Almost no households are employed at this level, and the minimum wage has virtually no impact on the economy. The minimum wage affects the economy through the wage markdown w/μ (lower panel of Figure 3). An increase in the minimum wage raises wage markdowns and lowers hiring rates. Even if the wage markdown exceeds 1, this does not necessarily mean that the hiring rate will immediately fall to zero. This is because the elasticity of labor substitution is finite at 5.545 and firms consider future productivity changes in determining hiring rates. However, raising the minimum wage to a binding level can be expected to significantly affect hiring and employment rates. An increase

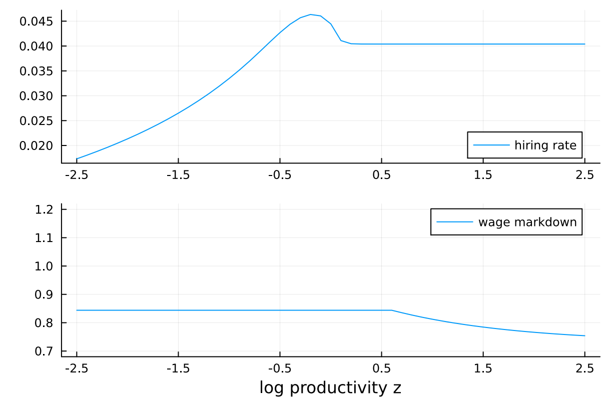

Figure 3: Hiring rates and wage markdowns

Note: The horizontal axis represents the log productivity of households, and the vertical axes represent the hiring rate and wage markdown w/μ, respectively.

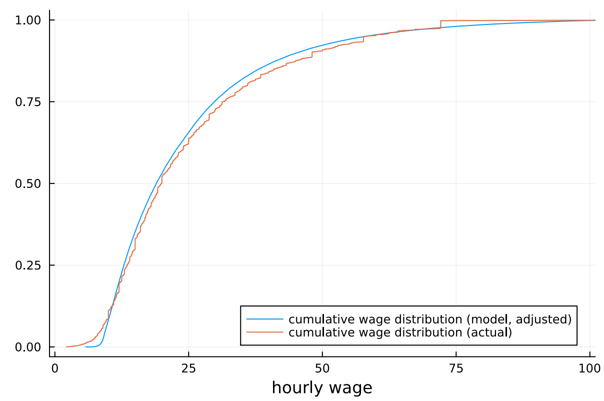

Figure 4: Wage distribution (model and actual)

Note: Wage distribution in the model (after calibration) and of the actual data. The latter is constructed by dividing weekly earnings by weekly hours worked using microdata from the 2019 CPS.

in the value of employment will lead to a transition from out-of-labor to employment via unemployment.

Figure 4 compares the wage distribution in the model and the actual wage distribution. The median of the model wage distribution is adjusted to match the median of the data ($19.95). Both distributions appear to be close to each other, except in the tails; however, this is unsurprising because the model reflects that the actual wage distribution can be approximated by a log-normal distribution. In the model, productivity follows a log-normal distribution; therefore, the wage distribution is similar to this distribution. Therefore, if the variances of both can be brought closer, and because they are included in the targets of the parameter calibration, the two distributions will nearly overlap.

This is consistent with the actual data showing that the minimum wage does not bind to the baseline calibration. There are not a few employed below the $7.25 federal minimum wage; hence, the baseline minimum wage is set to $2.13. Meanwhile, some deviations are observed in the right tail of the wage distribution.[12] Although a relatively large number of high-earnings households exist in the model, this does not affect the results because our

[12] The actual wage distribution shown in Figure 4 is constructed from the CPS. Atkinson et al. [2011] noted that the survey is not reliable for the higher income bracket because of limitations in sample size, and instances of incomplete or missing responses.

study focuses primarily on wages and employment for those with productivity below the median.

5 Minimum wage increase simulations

5.1 Steady states

Here, we examine the effects of minimum wage increases from the baseline of \$2.13 to \$7.25, \$10, \$12.5, and \$15 (MW7.25, MW10, MW12.5, and MW15, respectively). All parameters other than the minimum wage are held at their baseline values. Table 3 summarizes the impact on output, consumption, capital-output ratio, effective labor input, real interest rate, mean wage, median wage, labor participation rate, unemployment rate, social security benefits, tax revenue, corporate profits, government spending, Gini coefficient (wages, pre-tax income, after-tax income), social welfare, excess jobs, and missing jobs (defined below). The baseline scenario represents the standard U.S. economy, for which the parameter values are calibrated to match some of its moments.

Raising the minimum wage reduces the output, consumption, capital-output ratio, effective labor input, and labor participation rate. While this increases the value of employment and unemployment through an increase in wages and unemployment benefits, it also decreases hiring rates. The latter effect dominates and reduces the values of employment and unemployment, and the labor participation rate. Flinn [2006] indicated a possibility that a minimum wage increase raises the value of search, thereby increasing the labor participation rate and employment. However, his model and empirical results, which are similar to ours, also suggest that endogenous matching probabilities reduce employment.

The unemployment rate rises monotonically in response to a decrease in hiring rates owing to the minimum wage increase. Similarly, the mean wage increases monotonically as households with lower productivity move to out-of-labor. However, the median wage does not rise as much as the mean wage because the household productivity distribution has more mass in the middle than in the tails. Meanwhile, the real interest rate rises because a decline in the labor participation rate reduces household saving rates and investment.

Increasing the minimum wage to \$10 does not significantly affect the real economy. For example, the output decline is about 2% relative to the baseline, and the impact on consumption is negligible at -0.2%. However, the effects become significant when the minimum wage exceeds \$10. With a minimum wage of \$15, production declines by approximately 12% relative to the baseline, and consumption declines by 2.3%. The unemployment rate rises

Variables / Minimum wage	Baseline	MW7.25	MW10	MW12.5	MW15
Output ($Y_b = 100$)	100.0	99.9	98.0	93.6	88.2
Consumption ($Y_b = 100$)	63.3	63.3	63.1	62.5	61.8
Capital-output ratio ($K/12Y$)	3.00	2.99	2.98	2.96	2.94
Effective labor input ($L_b = 100$)	100.0	100.0	98.3	94.2	89.0
Real interest rate (%)	5.82	5.83	5.89	5.95	6.02
Mean wage ($)	26.8	26.8	27.4	29.1	31.0
Median wage ($)	19.9	19.9	19.8	21.3	22.8
Labor participation rate (%)	77.6	77.5	74.6	68.9	62.0
Unemployment rate (%)	5.04	5.08	5.62	7.85	9.65
Consumption equivalence ($\Delta c, Y_b = 100$)	0.00	0.21	1.07	2.05	3.10
Social security benefits ($\Sigma b, Y_b = 100$)	3.08	3.16	4.19	6.34	9.12
Tax ($Y_b = 100$)	11.3	11.3	11.2	11.0	10.7
Profit ($Y_b = 100$)	9.9	9.9	9.7	9.4	9.0
Gov't consumption ($Y_b = 100$)	18.1	18.0	16.7	14.1	10.6
Gini coefficient on wage rate	0.53	0.53	0.55	0.58	0.62
Gini coefficient on income before tax	0.51	0.51	0.52	0.55	0.58
Gini coefficient on disposal income	0.42	0.42	0.41	0.41	0.40
Missing jobs ($N_b + U_b = 100$)	0.00	0.11	3.26	10.2	16.9
Excess jobs ($N_b + U_b = 100$)	0.00	0.05	1.15	2.89	4.10

Table 3: Macroeconomic outcomes for each minimum wage level

Note: Y_b: baseline output, L_b: baseline effective labor, and $N_b + U_b$: baseline labor force

from 4.6 percentage points to 9.6%. Because the wage distribution is similar to the log-normal distribution that household productivity follows (see Figure 4) and low-productivity households at the left tail of the distribution enter the unemployment or out-of-labor states first, the impact increases nonlinearly.

Following Floden [2001], we measure the impact of a minimum wage increase on social welfare by consumption equivalence, defined as Δc satisfying the equation:

$$\frac{1}{\rho}\sum_h \int\int u(\hat{c}_{i,jh}, \gamma(h))didj = \frac{1}{\rho}\sum_h \int\int u(\hat{c}'_{i,jh} + \Delta c, \gamma(h))didj,$$

where $\hat{c}$ on the left-hand side denotes the optimal consumption in the baseline and $\hat{c}'$ on the right-hand side denotes the optimal consumption in each simulation case (precisely, h of nonworkers is also optimally chosen). A negative Δc implies that the transition to an alternative minimum wage level improves social welfare.[13] Table 3 shows that social welfare deteriorates monotonically in response to the minimum wage increase. However, even with the $15 minimum wage, the welfare loss is 3.1% of baseline output and not large because the decrease in consumption is moderate. Furthermore, as noted above, households that move to out-of-labor still gain utility from home production. Meanwhile, Berger et al. [2022] found that a minimum wage of $15.12 maximizes social welfare.[14] Under this optimal minimum wage, both non-high-school and high-school graduate households experience significant welfare improvements from higher wages exceeding 5% of the consumption equivalence. However, owners of capital experience a monotonic deterioration in their welfare due to the minimum wage increase because it lowers corporate profits and capital income, which is consistent with our study.

The consumption decline caused by the minimum wage hike is only minor compared with that in production because of the assumption that the income of unemployed and out-of-labor households is compensated for by social security benefits.[15] While social security benefits in the baseline are 3% of output, they increase to 9% under the $15 minimum wage. This difference is financed not by raising taxes but by cutting wasteful government spending. Thus, the simulation result shows the lower limit of social welfare deterioration due to a minimum wage increase. In reality, social welfare will be worse than this.

To evaluate the impact of the minimum wage increase on income inequality, we calculate the Gini coefficients. The Gini coefficient for wages increases monotonically in response to the minimum wage hike because households that worked for less than the minimum wage become unemployed or out-of-labor, relatively decreasing middle-income households. The same result applies to pretax income. Meanwhile, when measured in terms of disposable income after social security benefits, the Gini coefficient declines gradually, suggesting a reduction in inequality. This is because raising the minimum wage increases the total amount of social security benefits during unemployment spells by assumption, which implies a significant expansion of transfers to nonworkers as a trade-off.

Figure 5 illustrates the wage distribution for each minimum wage level. As in Figure 4,

[13] The social welfare defined here is based on the perspective of Benthamite utilitarianism because each household's utility is aggregated with equal weight.

[14] The measure of this social welfare is based on utilitarian principles, like ours. The authors also conducted an estimation based on Negishi weights, which yielded a significantly lower optimal minimum wage of $6.97.

[15] A lower employment rate leads to longer unemployment spells, as shown below, and requires greater social costs, including increased social security benefits.

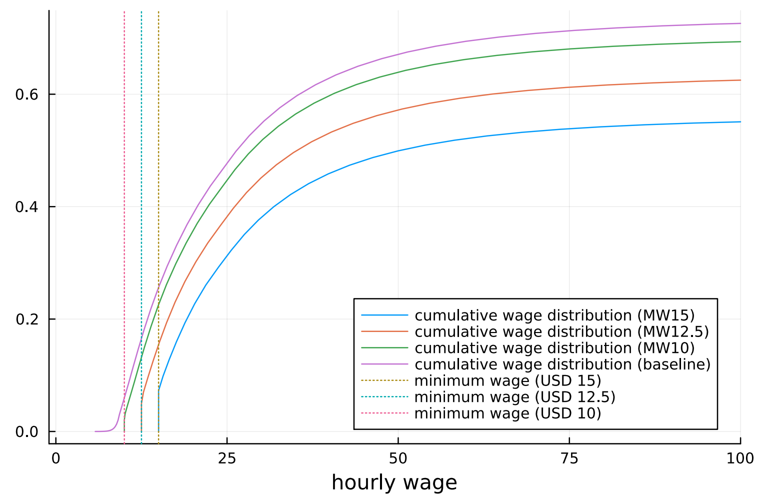

Figure 5: Wage distribution for each minimum wage level

Note: The horizontal axis represents the hourly wage level and the vertical axis represents the cumulative employment rate. The dashed vertical lines show each minimum wage level.

the horizontal and vertical axes represent the hourly wage level and cumulative employment rate, respectively. The lower bound of the distribution corresponds to the minimum wage level and raising it increases the employment bound to itself. Furthermore, a downward shift in the cumulative probability density distribution implies a contraction in total employment. As shown in Table 3, the employment ratio of 72% in the baseline falls to 52% at the $15 minimum wage.

Figure 6 shows the distribution of employment status for the baseline and $15 minimum wage, respectively, with productivity on the horizontal axis. The upper panel shows the distribution of employment and out-of-labor, where the right and left tails remain unchanged regardless of the minimum wage. Thus, as expected, the impact of the minimum wage on the distribution occurs only around the productivity level affected directly by it. Further, raising the minimum wage shifts the unemployment distribution to the right as shown in the bottom panel. This implies that the minimum wage increase raises the productivity thresholds between both out-of-labor and unemployment, and unemployment and employment. The top panel of Figure 7 shows the hiring rates in the baseline and $15 minimum wage cases, whereas the bottom panel shows the wage markdowns. Raising the minimum wage reduces employment opportunities for low-productivity households, which implies the existence of a mechanism that the wage markdowns increase when the minimum wage binds, worsening firm profitability, and thus, reducing hiring rates.

Figure 6: Distribution of employment, unemployment, and out-of-labor (baseline and \$15 minimum wage)

Note: The horizontal axis represents log productivity and the vertical axis represents the probability density for each status. The dashed vertical line shows the log productivity corresponding to the \$15 minimum wage.

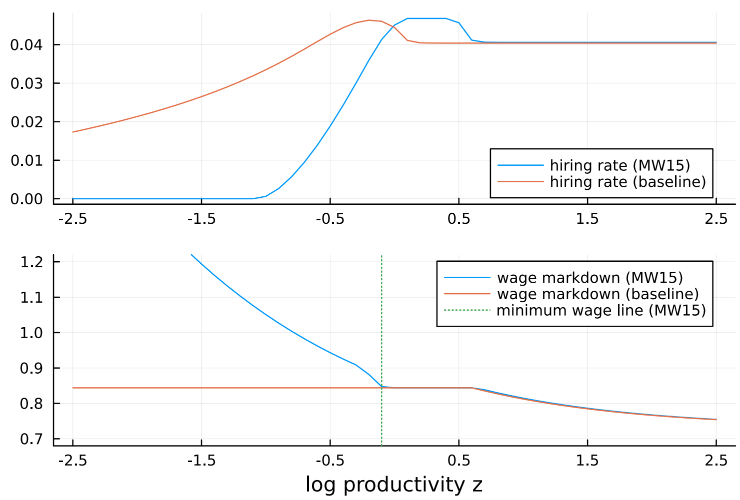

Figure 7: Hiring rates and wage markdowns (baseline and \$15 minimum wage)

Note: The horizontal axis represents log productivity. The dashed vertical line shows the log productivity corresponding to the \$15 minimum wage.

The left panel of Figure 8 shows the hourly wage distribution (probability density) at each minimum wage level. Cengiz et al. [2019] defined a decrease in the probability density

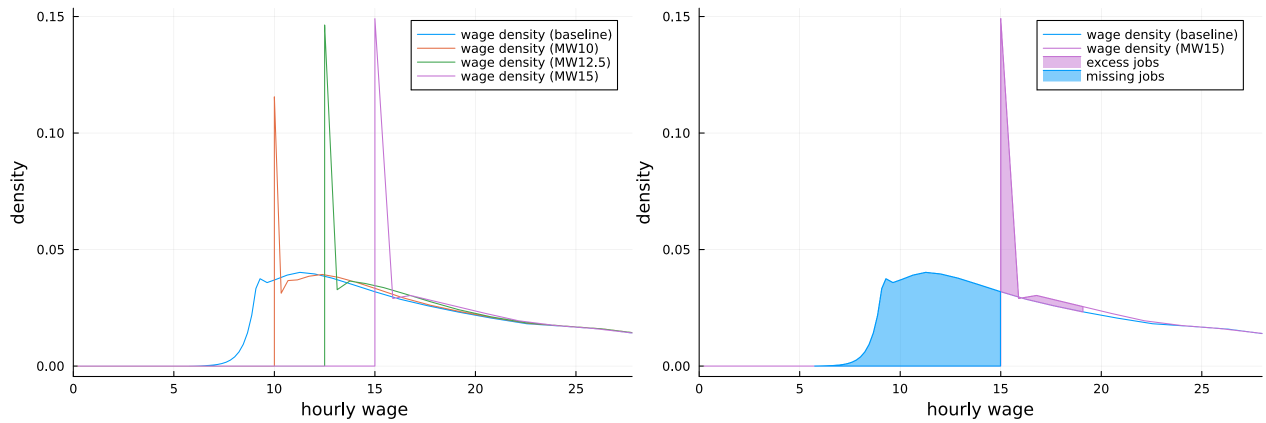

Figure 8: Wage distribution (simulation)

Note: The left panel shows the wage distribution for each minimum wage level. The right panel shows the wage distribution for the baseline and $15 minimum wage, where the blue shaded area represents missing jobs and pink shaded area represents excess jobs in response to the increase in the minimum wage from $2.13 (baseline) to $15.

of the hourly wage relative to the baseline as "missing jobs" and an increase as "excess jobs" (right panel of Figure 8). We demonstrate that they arise endogenously in our model, which Cengiz et al. [2019] found empirically. They exploited 138 cases of minimum wage increases, with an average of 10.1%, to observe employment effects using a DID approach. According to their estimates, missing jobs account for 1.8% of employment, while excess jobs up to +$4 from the new minimum wage are 2.1% and exceed the missing jobs. Thus, the authors concluded that a minimum wage hike increases employment.[16] The empirical fact that most excess jobs occur in bins above the new minimum wage is consistent with our results.

In our model, raising the minimum wage reduces hiring rates, thereby reducing employment, as argued above. For example, raising the minimum wage to $10 causes 3.2% missing jobs and 1.1% excess jobs, and the net employment decrease is 4% relative to the baseline labor force. Here, we define excess jobs as the sum of differences up to +$4 from the new minimum wage, following Cengiz et al. [2019]. Given that the case of $7.25 minimum wage is substantially identical to the baseline, the $10 minimum wage corresponds to a 38% increase. The net employment decreases in our study exceed the estimates of Cengiz et al. [2019]. This reflects that the events captured in their study are actual, and thus, the minimum wage in-

[16]In Cengiz et al. [2019], bins with an interval of $1 are created, and before and after treatment effects are compared. The authors identify the sum of the differences up to +$4 from the new minimum wage as excess jobs based on statistical significance.

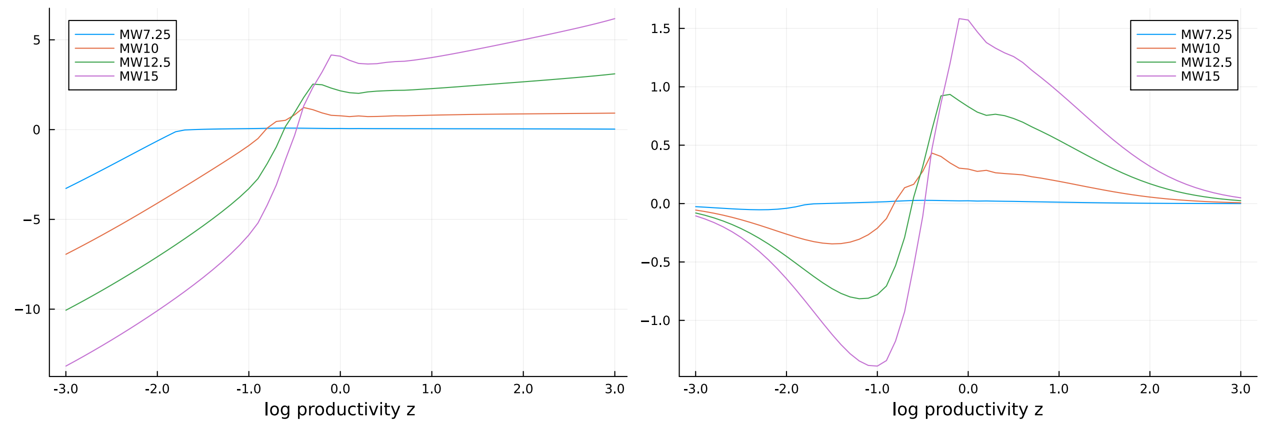

Figure 9: Consumption equivalent

Note: The figures illustrate the welfare changes relative to the baseline in response to the minimum wage increase, as measured by consumption equivalence. The left panel shows these albeit normalized by setting the baseline output to 100. The right panel shows them adjusted by multiplying the density of the productivity distribution so that the area represents the expected value.

creases are relatively small. Meanwhile, our model analysis suggests that minimum wage increases of more than 10% generate significant missing jobs relative to excess jobs. Missing and excess jobs increase monotonically as the minimum wage increases at 10.2% and 2.8%, respectively, with the \$12.5 minimum wage and 16.9% and 4.0%, respectively, with the \$15 minimum wage.

Figure 9 shows the changes in individual welfare compared with the baseline in response to the minimum wage increase, measured by consumption equivalence Δc_j for each productivity j, which we define using $\tilde{\Delta} c_{ijh}$ and $\tilde{\Delta} c'_{ijh}$ satisfying the following equation:

$$\frac{1}{\rho}\sum_h \int u(\hat{c}_{ijh} + \max(\tilde{\Delta} c'_j, 0), \gamma(h))di = \frac{1}{\rho}\sum_h \int u(\hat{c}'_{ijh} + \max(\tilde{\Delta} c'_j, 0), \gamma(h))di,$$

and letting $\Delta c_j = \max(\tilde{\Delta} c_{ijh}, 0) - \max(\tilde{\Delta} c'_{ijh}, 0)$. The positivity of consumption is ensured by dividing it into two terms. In Figure 9, the left panel shows the consumption equivalence for each productivity level (normalized by setting the baseline output to 100), whereas the right panel shows the consumption equivalence adjusted by multiplying the density of the productivity distribution so that the area represents the expected value. Social welfare decreases as the minimum wage increases, as shown in Table 3; however, the extent to which welfare improves or deteriorates depends on individual productivity. Raising the minimum wage to \$7.25 only affects individuals with low productivity in the left tail of the distribution,

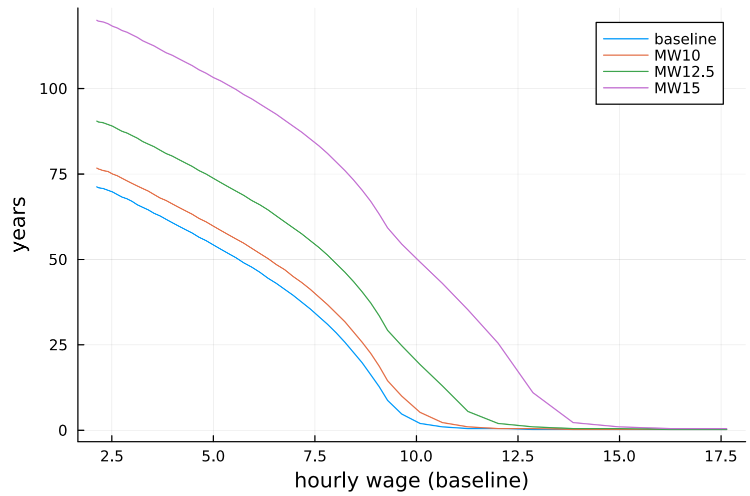

Figure 10: Duration of unemployment

Note: The vertical axis represents the number of years until the employment rate exceeds 0.5, conditional on being unemployed or out-of-labor in year 0. The horizontal axis represents productivity converted to the baseline wage level.

limiting welfare gains to a small portion of households. As the minimum wage increases, the upper limit of productivity at which welfare improves also shifts to the right, and the range within which the minimum wage binds expands. Meanwhile, a minimum wage increase lowers employed households' welfare with relatively high productivity through a decline in job-finding rates. Meanwhile, when missing and excess jobs in Figure 8 arise in a particular region directly affected by the minimum wage, the welfare improvements and losses in Figure 9 are more widespread.

Figure 10 illustrates the number of years until the employment rate exceeds 0.5, conditional on being unemployed or out-of-labor at $t = 0$. Productivity is on the horizontal axis (converted to the baseline wage level). Since hiring rates decline as the minimum wage increases (see the top panel of Figure 7), unemployment spells (including being out-of-labor) become prolonged, especially for individuals with lower initial productivity. As the productivity of each household varies following Equation (1), the transition to employment is primarily driven by their productivity increase. Note that regardless of the initial productivity, the individual employment rate equals the steady-state employment rate (see Table 3) after a sufficient period. Thus, raising the minimum wage extends the duration of unemployment spells in two ways: first, as already noted, it lowers hiring rates, and second, it lowers the steady-state employment rate. The prolonged unemployment spells incur substantial social costs both within the model and in the actual economy.

5.2 Dynamics analysis

We examine the transition dynamics of the minimum wage increase to analyze a more realistic scenario. The average inflation rate over the past 30 years has been approximately 2.5% in the U.S., while the monetary policy inflation target is 2%. As the minimum wage is legislated in nominal terms, its real value is eroded by inflation unless it is raised proportionately. Figure 11 shows the federal minimum wage and its real values deflated by the CPI-U. While the federal minimum wage has been raised every few years, from $0.75 in 1950 to $7.25 in 2010, the real minimum wage has hovered around $8 in 2019 prices.

To examine the dynamic impact on macroeconomic outcomes, we simulate a scenario in which the minimum wage is raised to $15 and then left unchanged. The initial condition (year 0) is the steady state in the baseline ($2.13 minimum wage). The minimum wage is raised to $15 in year 2, followed by a 2% depreciation (upper panel of Figure 12). The terminal condition is the real minimum wage of $7.25, which is the current federal minimum wage. As the difference between the minimum wages of $2.13 and $7.25 is small in the model (see Table 3), the initial and terminal conditions can be considered identical. The expectations are assumed to have perfect foresight (assuming an MIT shock). That is, each economic agent makes decisions at each time point after year 1, given the paths of the exogenous variable (the real minimum wage) and macro endogenous variables (wages, real interest rates, etc.) up to the steady state thereafter. Because the model is described in continuous time, the calculations are implemented after discretization with the unit of time being a month.

Figures 12 to 14 show the transition paths of the macroeconomic variables in response to the minimum wage increase. After it is raised to $15 in year 2, its real value steadily depreciates by 2% inflation until it reaches $7.25 in year 38. The mean wage peaks at $30.3 in year 6 and then declines, returning to nearly its initial value. The mean wage increase is attributed to a composition effect resulting from the loss of low-wage employment. By the time the real minimum wage reaches the terminal condition of $7.25, the other variables return to around their initial values.

The employment ratio falls by approximately 12 percentage points from 74% in year 6 and gradually returns to its initial level. Further, the ratio of unemployment rises from 4% to 13% in year 6, while that of out-of-labor rises from 22% to a peak of 25% in year 9. The employment dynamics are discussed in more detail below. Output drops by 5 percentage

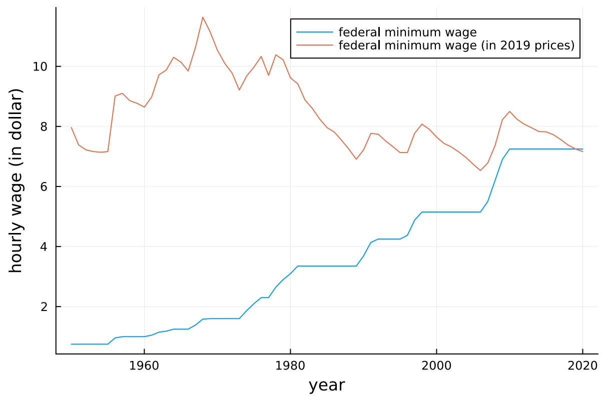

Figure 11: Nominal and real federal minimum wages

Note: The real federal minimum wage (in 2019 prices) is calculated by deflating the nominal wage by the CPI-U.

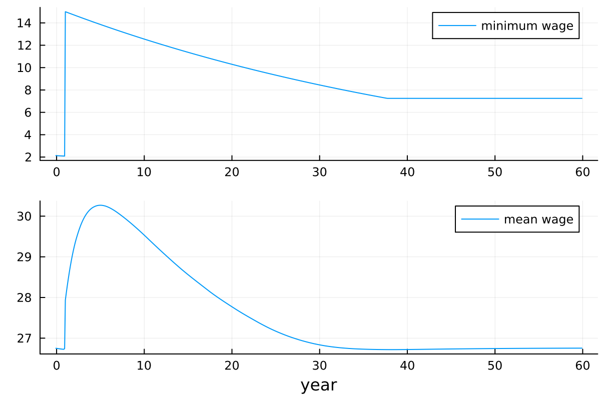

Figure 12: Minimum and mean wages

Note: The real minimum wage in the top panel is set exogenously according to the simulation scenario. The bottom panel shows the transition path of the mean wage. The vertical axes are in dollars.

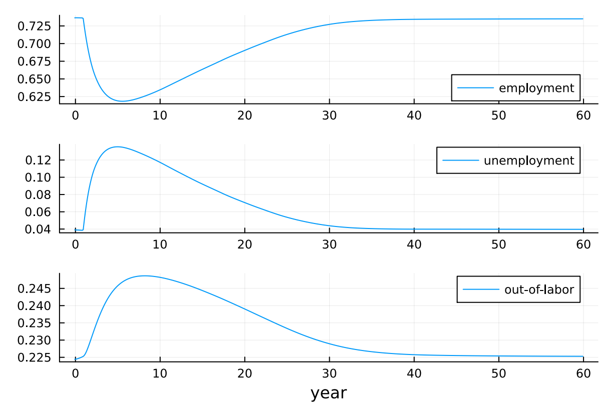

Figure 13: Employment status (transition path)

Note: Transition paths for employment (top panel), unemployment (middle panel), and out-of-labor ratios (bottom panel) following the $15 temporary minimum wage change. The vertical axes represent each state's ratio to population (= 1).

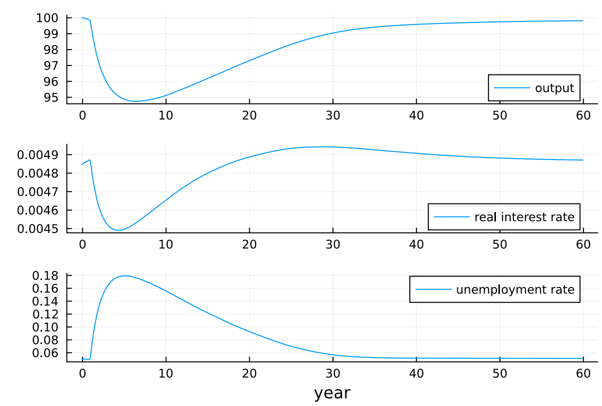

Figure 14: Output, real interest rate, and unemployment rate

Note: Transition paths for output (top panel), the real interest rate (middle panel), and the unemployment rate (bottom panel) following the $15 temporary minimum wage change.

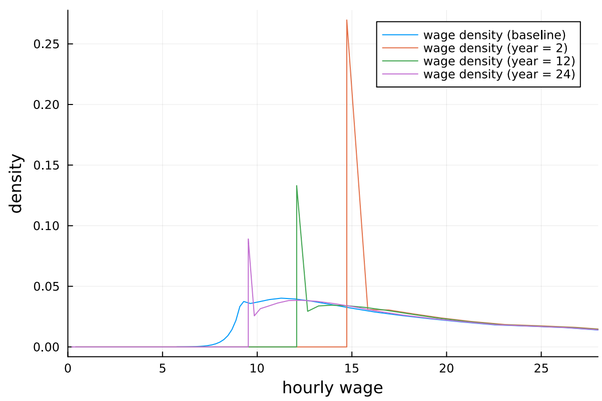

Figure 15: Wage distribution (transition path)

Note: Wage distribution at each period along the transition path following the $15 temporary minimum wage change.

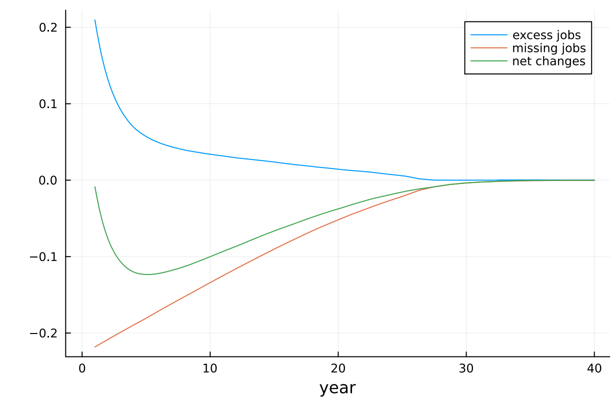

Figure 16: Excess jobs, missing jobs, and net changes

Note: Transition paths for excess jobs, missing jobs, and net employment changes following the $15 temporary minimum wage change.

points by year 7, bottoms out, and then gradually returns to its initial value. This reflects that a minimum wage increase raises labor costs, thereby reducing effective labor input and capital stock. The real interest rate also falls to 4.5% in year 5. In the steady state with the $15 minimum wage, the real interest rate rises by 0.2 percentage points to 6.0% relative to the baseline, as shown in Table 3. However, it falls on the transition path because capital stock takes time to adjust and a temporary excess of capital stock occurs.[17]

The unemployment rate rises to 18% in year 6, well above the steady-state level of 9.6% with the $15 minimum wage. This is because firms reserve employment anticipating a decline in the real minimum wage, and households actively seek jobs anticipating future labor demand increases. Regarding the latter, the increase in the value of employment because of a higher minimum wage keeps them in the labor force rather than letting them choose out-of-labor.

Figure 15 illustrates the hourly wage distribution at each point along the transition path. As expected, the numbers of missing and excess jobs peak immediately after the minimum wage is raised to $15 in year 2, and are significantly larger than those in the steady state

[17]Hurst et al. [2022] also examined the dynamic analysis of the minimum wage increase. They included putty-clay capital in the model and found limited job losses in the short run, because of the lack of capital adjustment; however, in the long run, labor is substituted for capital, thereby reducing employment for low-skilled, non-college workers.

with the \$15 minimum wage. Subsequently, as the real minimum wage declines, the binding wage level shifts to the left, thereby reducing both missing and excess jobs. Figure 16 shows the time-series variation in missing and excess jobs. Notably, the net changes – that is, the difference between missing and excess jobs – are relatively small in absolute value immediately after the minimum wage increase. This suggests that observing an employment decline may be difficult shortly after a minimum wage increase in natural experiments.

6 Discussions

Here, we examine how our model, its underlying assumptions, and derived insights differ from those of relevant studies focusing on welfare outcomes. In our model, the matching between households and firms is efficient as we have no mismatch; therefore, raising the minimum wage does not improve welfare. By contrast, other studies (e.g., Dustmann et al. [2022]) suggest that raising the minimum wage can improve economic efficiency through the reallocation effect.

A sufficient condition for the reallocation effect is that firms have monopsony power. However, to meet this condition, the elasticity of substitution of labor supply across firms must be low if households are rational and have free movement of labor. To address this, Berger et al. [2022] and Hurst et al. [2022] introduced the assumption of the family; consequently, households prefer supplying labor to a wide variety of firms. This assumption not only seems unrealistic but also underestimates the role of social security systems, including the minimum wage.

Dustmann et al. [2022] attributed the reallocation effect to the minimum wage increase shifting low-income households from small to large firms. Our model assumes perfect competition and homogeneity across firms to combine the heterogeneous agent model with an undirected search-matching model. In reality, if low-productivity firms retain low-income workers because of policies favoring small and medium-sized enterprises, a higher minimum wage will improve the efficiency of matching between households and firms, and mitigate the adverse effect on employment. Our model may overestimate the negative effects of minimum wage increases by not accounting for the reallocation effect.

For example, if firms randomly offer wages in the matching process, as in Flinn [2006] and Engbom and Moser [2022], social welfare can be improved by raising the minimum wage. However, in our model, wages are deterministically determined by Nash bargaining between households and firms. If we assume firm heterogeneity and that matched wages are randomly determined, we may observe results similar to those of Dustmann et al. [2022] and Engbom and Moser [2022]. However, larger minimum wage increases will affect firms with moderate productivity and reduce employment, as suggested in our study. Moreover, raising the minimum wage is not the only effective solution for addressing mismatches. If matching inefficiencies exist among low-income workers, they are also likely to exist among higher-income workers. However, they cannot be reallocated by the minimum wage.

7 Conclusion

We construct a model with heterogenous household productivity in which the unemployment rate is endogenously determined to explore the impact of a large unprecedented minimum wage increase. We calibrated the model parameters to replicate the U.S. economy and simulated the impact of the minimum wage increase.

Our theoretical contribution includes constructing a model in which the employment status of each household is determined by a search-matching framework without relying on the assumption of a representative household or family. We find that a minimum wage increase applied to the entire area of free labor mobility has a broad impact on macroeconomic outcomes, including employment. The results of the dynamic simulation suggest that the net job losses immediately after a minimum wage increase are small. This has important implications for the appropriate design of empirical analyses using natural experiments.

Our model has several limitations. Without the reallocation effect, a minimum wage increase does not improve social welfare. For example, the effect occurs if the matched wage is uncertain. To address this issue, firm heterogeneity is necessary and the impact of minimum wage increases on employment should be investigated through firm behavior. We leave this analysis to future works.

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