

# 式展開の補足（ノート2：最適成長モデル）

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## ラグランジアン

$$\begin{aligned}\Lambda &= \sum_{i=t}^{\infty} \beta^{i-t} \left[ \ln(C_i) + \lambda_i \left\{ r_i K_i + w_i - C_i + (1 - \delta)K_i - K_{i+1} \right\} \right] \\ &= \left[ \ln(C_t) + \lambda_t \left\{ r_t K_t + w_t - C_t + (1 - \delta)K_t - K_{t+1} \right\} \right] \\ &\quad + \beta \left[ \ln(C_{t+1}) + \lambda_{t+1} \left\{ r_{t+1} K_{t+1} + w_{t+1} - C_{t+1} + (1 - \delta)K_{t+1} - K_{t+2} \right\} \right] \\ &\quad + \beta^2 \left[ \ln(C_{t+2}) + \lambda_{t+2} \left\{ r_{t+2} K_{t+2} + w_{t+2} - C_{t+2} + (1 - \delta)K_{t+2} - K_{t+3} \right\} \right] \\ &\quad + \dots\end{aligned}\tag{1}$$

ラグランジアンを  $C_t$  で偏微分すると、青字で書いた項以外は消える。

$$\begin{aligned}\Lambda = & \left[ \ln(C_t) + \lambda_t \left\{ r_t K_t + w_t - C_t + (1 - \delta)K_t - K_{t+1} \right\} \right] \\ & + \beta \left[ \ln(C_{t+1}) + \lambda_{t+1} \left\{ r_{t+1} K_{t+1} + w_{t+1} - C_{t+1} + (1 - \delta)K_{t+1} - K_{t+2} \right\} \right] \\ & + \beta^2 \left[ \ln(C_{t+2}) + \lambda_{t+2} \left\{ r_{t+2} K_{t+2} + w_{t+2} - C_{t+2} + (1 - \delta)K_{t+2} - K_{t+3} \right\} \right] \\ & + \dots\end{aligned}\tag{2}$$

$$\begin{aligned}\frac{\partial \Lambda}{\partial C_t} &= \frac{\partial}{\partial C_t} \left[ \ln(C_t) - \lambda_t C_t \right] = 0 \\ \Leftrightarrow \frac{1}{C_t} - \lambda_t &= 0\end{aligned}\tag{3}$$

ラグランジアンを  $K_{t+1}$  で偏微分すると、青字で書いた項以外は消える。

$$\begin{aligned}
 \Lambda = & \left[ \ln(C_t) + \lambda_t \left\{ r_t K_t + w_t - C_t + (1 - \delta)K_t - K_{t+1} \right\} \right] \\
 & + \beta \left[ \ln(C_{t+1}) + \lambda_{t+1} \left\{ r_{t+1} K_{t+1} + w_{t+1} - C_{t+1} + (1 - \delta)K_{t+1} - K_{t+2} \right\} \right] \\
 & + \beta^2 \left[ \ln(C_{t+2}) + \lambda_{t+2} \left\{ r_{t+2} K_{t+2} + w_{t+2} - C_{t+2} + (1 - \delta)K_{t+2} - K_{t+3} \right\} \right] \\
 & + \dots
 \end{aligned} \tag{4}$$

$$\begin{aligned}
 \frac{\partial \Lambda}{\partial K_{t+1}} &= \frac{\partial}{\partial K_{t+1}} \left[ -\lambda_t K_{t+1} + \beta \lambda_{t+1} (r_{t+1} + 1 - \delta) K_{t+1} \right] = 0 \\
 \Leftrightarrow & \beta (r_{t+1} - \delta + 1) \lambda_{t+1} - \lambda_t = 0
 \end{aligned} \tag{5}$$

ラグランジアンを  $C_{t+1}$  で偏微分すると、青字で書いた項以外は消える。

$$\begin{aligned}\Lambda = & \left[ \ln(C_t) + \lambda_t \left\{ r_t K_t + w_t - C_t + (1 - \delta)K_t - K_{t+1} \right\} \right] \\ & + \beta \left[ \ln(C_{t+1}) + \lambda_{t+1} \left\{ r_{t+1} K_{t+1} + w_{t+1} - C_{t+1} + (1 - \delta)K_{t+1} - K_{t+2} \right\} \right] \\ & + \beta^2 \left[ \ln(C_{t+2}) + \lambda_{t+2} \left\{ r_{t+2} K_{t+2} + w_{t+2} - C_{t+2} + (1 - \delta)K_{t+2} - K_{t+3} \right\} \right] \\ & + \dots\end{aligned}\tag{6}$$

$$\begin{aligned}\frac{\partial \Lambda}{\partial C_{t+1}} &= \frac{\partial}{\partial C_{t+1}} \beta \left[ \ln(C_{t+1}) - \lambda_{t+1} C_{t+1} \right] = 0 \\ \Leftrightarrow \frac{\partial}{\partial C_{t+1}} \left[ \ln(C_{t+1}) - \lambda_{t+1} C_{t+1} \right] &= 0 \\ \Leftrightarrow \frac{1}{C_{t+1}} - \lambda_{t+1} &= 0\end{aligned}\tag{7}$$

ラグランジアンを  $K_{t+2}$  で偏微分すると、青字で書いた項以外は消える。

$$\begin{aligned}\Lambda = & \left[ \ln(C_t) + \lambda_t \left\{ r_t K_t + w_t - C_t + (1 - \delta)K_t - K_{t+1} \right\} \right] \\ & + \beta \left[ \ln(C_{t+1}) + \lambda_{t+1} \left\{ r_{t+1} K_{t+1} + w_{t+1} - C_{t+1} + (1 - \delta)K_{t+1} - K_{t+2} \right\} \right] \\ & + \beta^2 \left[ \ln(C_{t+2}) + \lambda_{t+2} \left\{ r_{t+2} K_{t+2} + w_{t+2} - C_{t+2} + (1 - \delta)K_{t+2} - K_{t+3} \right\} \right] \\ & + \dots\end{aligned}\tag{8}$$

$$\begin{aligned}\frac{\partial \Lambda}{\partial K_{t+2}} &= \frac{\partial}{\partial K_{t+2}} \left[ -\beta \lambda_{t+1} K_{t+2} + \beta^2 \lambda_{t+2} (r_{t+2} + 1 - \delta) K_{t+2} \right] = 0 \\ \Leftrightarrow & \beta (r_{t+2} - \delta + 1) \lambda_{t+2} - \lambda_{t+1} = 0\end{aligned}\tag{9}$$

以下同様