式展開の補足（ノート2：最適成長モデル）

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ラグランジアン

$$
\begin{aligned}
\Lambda &= \sum_{i=t}^{\infty} \beta^{i-t} \Big[\ln(C_i) + \lambda_i \Big\{ r_i K_i + w_i - C_i + (1-\delta) K_i - K_{i+1} \Big\} \Big] \\
&= \Big[\ln(C_t) + \lambda_t \Big\{ r_t K_t + w_t - C_t + (1-\delta) K_t - K_{t+1} \Big\} \Big] \\
&+ \beta \Big[\ln(C_{t+1}) + \lambda_{t+1} \Big\{ r_{t+1} K_{t+1} + w_{t+1} - C_{t+1} + (1-\delta) K_{t+1} - K_{t+2} \Big\} \Big] \\
&+ \beta^2 \Big[\ln(C_{t+2}) + \lambda_{t+2} \Big\{ r_{t+2} K_{t+2} + w_{t+2} - C_{t+2} + (1-\delta) K_{t+2} - K_{t+3} \Big\} \Big] \\
&+ \dots
\end{aligned} \tag{1}
$$

ラグランジアンを C_t で偏微分すると、青字で書いた項以外は消える。

$$
\begin{aligned}
\Lambda = &\Big[\ln(C_t) + \lambda_t\Big\{r_t K_t + w_t - C_t + (1-\delta)K_t - K_{t+1}\Big\}\Big] \\
&+ \beta\Big[\ln(C_{t+1}) + \lambda_{t+1}\Big\{r_{t+1} K_{t+1} + w_{t+1} - C_{t+1} + (1-\delta)K_{t+1} - K_{t+2}\Big\}\Big] \\
&+ \beta^2\Big[\ln(C_{t+2}) + \lambda_{t+2}\Big\{r_{t+2} K_{t+2} + w_{t+2} - C_{t+2} + (1-\delta)K_{t+2} - K_{t+3}\Big\}\Big] \\
&+ \dots
\end{aligned}
\tag{2}
$$

$$
\begin{aligned}
&\frac{\partial \Lambda}{\partial C_t} = \frac{\partial}{\partial C_t}\Big[\ln(C_t) - \lambda_t C_t\Big] = 0 \\
&\Leftrightarrow \frac{1}{C_t} - \lambda_t = 0
\end{aligned}
\tag{3}
$$

ラグランジアンを K_{t+1} で偏微分すると、青字で書いた項以外は消える。

$$
\begin{aligned}
\Lambda = & \left[\ln(C_t) + \lambda_t \left\{ r_t K_t + w_t - C_t + (1-\delta)K_t - K_{t+1} \right\}\right] \\
& + \beta \left[\ln(C_{t+1}) + \lambda_{t+1} \left\{ r_{t+1} K_{t+1} + w_{t+1} - C_{t+1} + (1-\delta)K_{t+1} - K_{t+2} \right\}\right] \\
& + \beta^2 \left[\ln(C_{t+2}) + \lambda_{t+2} \left\{ r_{t+2} K_{t+2} + w_{t+2} - C_{t+2} + (1-\delta)K_{t+2} - K_{t+3} \right\}\right] \\
& + \dots
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
& \frac{\partial \Lambda}{\partial K_{t+1}} = \frac{\partial}{\partial K_{t+1}} \left[-\lambda_t K_{t+1} + \beta \lambda_{t+1} \left(r_{t+1} + 1 - \delta \right) K_{t+1} \right] = 0 \\
& \Leftrightarrow \beta \left(r_{t+1} - \delta + 1 \right) \lambda_{t+1} - \lambda_t = 0
\end{aligned}
\tag{5}
$$

ラグランジアンを C_{t+1} で偏微分すると、青字で書いた項以外は消える。

$$
\begin{aligned}
\Lambda = & \Big[\ln(C_t) + \lambda_t \Big\{ r_t K_t + w_t - C_t + (1-\delta)K_t - K_{t+1} \Big\}\Big] \\
& + \beta \Big[\ln(C_{t+1}) + \lambda_{t+1} \Big\{ r_{t+1} K_{t+1} + w_{t+1} - C_{t+1} + (1-\delta)K_{t+1} - K_{t+2} \Big\}\Big] \\
& + \beta^2 \Big[\ln(C_{t+2}) + \lambda_{t+2} \Big\{ r_{t+2} K_{t+2} + w_{t+2} - C_{t+2} + (1-\delta)K_{t+2} - K_{t+3} \Big\}\Big] \\
& + \dots
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
& \frac{\partial \Lambda}{\partial C_{t+1}} = \frac{\partial}{\partial C_{t+1}} \beta \Big[\ln(C_{t+1}) - \lambda_{t+1} C_{t+1}\Big] = 0 \\
\Leftrightarrow & \frac{\partial}{\partial C_{t+1}} \Big[\ln(C_{t+1}) - \lambda_{t+1} C_{t+1}\Big] = 0 \\
\Leftrightarrow & \frac{1}{C_{t+1}} - \lambda_{t+1} = 0
\end{aligned}
\tag{7}
$$

ラグランジアンを K_{t+2} で偏微分すると、青字で書いた項以外は消える。

$$
\begin{aligned}
\Lambda = &\left[\ln(C_t) + \lambda_t\left\{r_t K_t + w_t - C_t + (1-\delta)K_t - K_{t+1}\right\}\right] \\
&+ \beta\left[\ln(C_{t+1}) + \lambda_{t+1}\left\{r_{t+1}K_{t+1} + w_{t+1} - C_{t+1} + (1-\delta)K_{t+1} - K_{t+2}\right\}\right] \\
&+ \beta^2\left[\ln(C_{t+2}) + \lambda_{t+2}\left\{r_{t+2}K_{t+2} + w_{t+2} - C_{t+2} + (1-\delta)K_{t+2} - K_{t+3}\right\}\right] \\
&+ \dots
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
&\frac{\partial\Lambda}{\partial K_{t+2}} = \frac{\partial}{\partial K_{t+2}}\left[-\beta\lambda_{t+1}K_{t+2} + \beta^2\lambda_{t+2}\left(r_{t+2} + 1 - \delta\right)K_{t+2}\right] = 0 \\
&\Leftrightarrow \beta\left(r_{t+2} - \delta + 1\right)\lambda_{t+2} - \lambda_{t+1} = 0
\end{aligned} \tag{9}
$$

以下同様